Large Scale Simulation of Magnetization Process of HTS Undulator for X-ray FEL Based on T-method

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In a development of the next generation light source, a compact X-ray Free Electron Laser (X-FEL), it is necessary to achieve stronger undulator magnetic field with compact magnets. For this purpose, the Pure-type High-Tc Superconductor (HTS) undulator was proposed. Then, since it is very difficult to adjust the magnetic field distribution after the HTS magnets are magnetized all together in a cryostat, a very careful design of the undulator is required by using a high accuracy numerical analysis of magnetic field distribution in the design stage. For this reason, authors have been working on a development of a numerical simulation code of the magnetization process of the HTS undulator by using the current vector potential method (T-method). In this work, the previously developed numerical code is modified to be used in much larger scale simulation for a practical FEL design.

*Index Terms***—Free Electron Laser (FEL), Bulk HTS, Undulators, Current vector potential method (T-method), Critical state model.**

I. INTRODUCTION

The X-ray Free Electron Laser (X-FEL) provides us a high-The X-ray Free Electron Laser (X-FEL) provides us a high-
intensity and coherent X-ray, which can be applied to many advanced technologies such as structural analysis of proteins, developments of nano-materials, and so on. However, the X-FEL is now available only in a few big laboratories such as SPring-8 in the world [1] since the X-FEL machine is a very large system. Accordingly, it is not sufficient for being widely used. Then, it is essential to develop a compact size machine for being much more widely used. For achievement of the small size X-FEL, it is known that the FEL undulator has to be constructed by a short period and high-intensity magnets. Then, the High-Tc Superconductor (HTS) magnet has been considered to be used for the strong magnet undulator [2]-[3]. However, it is very difficult to adjust the positions of the individual HTS magnet after it is changed to a superconducting state inside a cryostat. Consequently, a numerical simulation of the magnetization process of the HTS plays very important role for determining a suitable magnet size and alignment in the machine design stage. We have been working on a development of a numerical simulation code for the magnetization process of the HTS undulator based on the current vector potential method (T-method) [4]-[5] combining with the critical state model for the shielding current in the HTS [6]-[7], and confirm a sufficiently good agreement between the simulation result and its measurement data of magnetic field distribution for three magnets Pure-type HTS undulator. In general, the X-FEL undulator consists of more than two hundred magnets, and therefore the much larger scale simulation is required for the practical use of the developed code to the real X-FEL. This paper presents a modified simulation scheme of the HTS magnetization process for the

large scale simulation of the Pure-type HTS undulator.

II.PURE-TYPE SUPERCONDUCTING UNDULATOR

Fig. 1 shows an overview of the Pure-type HTS undulator. Many thin rectangular shape HTSs are arranged with the direction of electron beam motion (*x*-axis) [3]. After the HTS undulator is changed to superconducting state all together in the cryostat, circulating horizontal shielding currents are induced in the individual HTS by applying time dependent vertical magnetic field **B**₀. Then, alternative vertical magnetic field is formed along the *x*-axis by the periodic shielding currents in HTSs, and the electron would perform undulator motion as in a typical FEL. It is known that a precise sinusoidal distribution of the vertical magnetic field along *x*direction is required for the normal FEL operation, therefore, it is necessary to carefully design the magnet alignment by using the numerical simulation of the magnetization process of the HTS undulator in the machine design stage since it is very difficult to adjust the individual HTS position in the cryostat after the magnetization.

III. SIMULATION OF MAGNETIZATION PROCESS OF PURE-TYPE HTS UNDULATOR BASED ON T-METHOD

We here use the current vector potential method combing with the critical state model on shielding current for the simulation of the magnetization process of the HTS [4]-[7]. In the T-method, the shielding current **J** induced in the HTS is expressed as $J = \nabla \times T$ by using the current vector potential T. The governing equation for the **T** is the following integrodifferential equation,

HTSFig. 1. Overview of Pure-type undulator *^x*

$$
\nabla \times \frac{1}{\sigma} \nabla \times \mathbf{T} + \mu_0 \frac{\partial \mathbf{T}}{\partial t} + \frac{\mu_0}{4\pi} \int_s \frac{\partial \mathbf{T} \cdot \mathbf{n}}{\partial t} \nabla \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) dS' = -\frac{\partial \mathbf{B}_0}{\partial t},\tag{1}
$$

where, σ is conductivity, μ_0 is permeability, S is the surface of the HTS, **n** is a unit normal vector on S, and \mathbf{B}_0 is externally applied magnetic field. Then the current vector potential has to satisfy the following boundary and gauge conditions respectively, $\mathbf{T} \times \mathbf{n} = 0$ on S, $\nabla \cdot \mathbf{T} = 0$ in domain. It is assumed that the shielding current is induced in *x*-*y* horizontal plane, then the HTS can be expressed by the 2D thin-plate model, and the current vector potential has only the *z*-component. In addition, Ohm's law is modified to the following critical current model for describing the shielding current behavior in the HTS,

$$
\mathbf{J} = J_c(|\mathbf{B}|)\frac{\mathbf{E}}{|\mathbf{E}|} \text{ if } |\mathbf{E}| \neq 0, \quad \frac{\partial \mathbf{J}}{\partial t} = 0 \text{ if } |\mathbf{E}| = 0. \tag{2}
$$

where J_c is the critical current of the HTSs and **E** is the electric field. We here use Bean's model $J_c(|\mathbf{B}|) = J_c$ = const. for the critical current. Then, to appropriately evaluate interaction between the shielding currents in HTSs, magnetic fields created by all other HTSs are included in the right-hand side of (1) as an external magnetic field in addition to externally applied field \mathbf{B}_0 (see Fig.2). In particular, the calculation of interaction from all other HTSs becomes a hot-spot of an entire calculation when the number of HTSs in the undulator is very large.

IV. NUMERICAL EXAMPLES

In a real FEL machine, the number of the undulator magnets is typically about 280 [1], which means straightforward use of conventional simulation code results in very large calculation. For reducing computation time in the large scale simulation, we consider to partially omit the interactions from other HTSs which are sufficiently far from considered HTS, so that the contribution from the far distance HTSs can be neglected.

The first numerical example is ten HTSs undulator indicated in Fig. 1. The size of individual HTS is taken to be $10 \text{mm} \times 15 \text{mm} \times 4 \text{mm}$, and J_c is $7 \times 10^8 [\text{A/m}^2]$ for all HTSs. Fig. 3 indicates vertical magnetic field distributions beyond on the undulator after the magnetization for the cases that no interaction from the nearest one, two, and three magnets are taken into account in the external field term of (1), compared with the full interaction simulation. It is found from Fig. 3 that the nearest three magnets should be taken into account for the calculation of the interaction from other HTSs. Then,

maximum relative error of the vertical magnetic field compared with the full interaction calculation is about 0.9%. Fig. 4 indicates a vertical magnetic field distribution after magnetization for a hundred HTSs undulator in which the nearest three magnets are included in the interaction calculation, as an example of large scale simulation. We can see that the stable simulation of the magnetization process was obtained. The calculation time of Fig. 4 was about 50 hours by core i7 PC, which is about 10 times faster than about 20 days of an estimated full interaction simulation.

V.CONCLUSION

This paper has presented a modified scheme for reducing calculation time in the large scale simulation of the magnetization process in the Pure-type HTS undulator. To carefully evaluate the calculation of the interaction between HTSs in (1), it was found that the interaction from the nearest three magnets were sufficient for accuracy of less than 1% relative error and calculation time can be effectively reduced. On the other hand, real FEL undulator has about 280 magnets which is 3 times bigger size than that of Fig. 4. For the real size simulation, memory size of the code should be also reduced by any other effective approximations, which will be a near future work.

REFERENCES

- [1] Y Otake, "Overview of SACLA Machine Status," *Proceedings of LINAC2012*, pp. 427-431, 2012.
- [2] T Kii and R Kinjo, "Application of bulk high-temperature superconduc tor to insertion device for synchrotron radiation," *J. Cryo. Soc. Jpn.*, vol.46, pp. 118-124, 2011.
- [3] T Tanaka, R Tsuru and H Kitamura, "Pure-type superconductor permanent magnet undulator," *J. Synchrotron Radiat.*, vol. 12, pp. 442-447, 2005.
- [4] M Tsuchimoto, K Demachi, I Itoh, "Numerical evaluation of uniform magnetic field within superconducting swiss roll," *Physica C*, vol. 412- 414, pp.719-722, 2004.
- [5] M Tsuchimoto, "Numerical evaluation of field profile in an undulator with bulk HTS," *Phys. Procedia*, vol. 27, 452-455, 2012.
- [6] Yi Deri, H Kawaguchi, M Tsuchimoto, T. Tanaka, "Simulation of magnetization process of pure-type superconductor magnet undulator based on T-method," *Physica C*, vol. 518, pp. 106-110, 2015.
- [7] Yi Deri, F Tanaka, H Kawaguchi, M Tsuchimoto, "Design of HTSC undulator magnet array based on simulation of magnetization process," *JASSE,* vol. 2, pp. 34-43, 2015. Allother magnets

Bz[T]

The nearest $\overline{3}$ magnets (Error = 0.9%)

6.0E-01

No interaction

Fig. 4. Distribution of vertical magnetic field component along hundred HTSs undulator